# Chapter 9
# Measurement Bias

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# Measurement error

- All variables were perfectly measured $\rightarrow$ unrealistic assumption!
- Pedestrian example) what if recorder of the pedestrians' responses made many mistakes?
- Various types of measurement error can be generated

# Measurement error

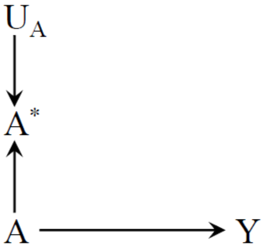


**Figure 9.1**

- $A^*$: measured treatment
- $U_A$: **measurement error** for $A$
    - all factors other than $A$ that determine the value of $A^*$

# Measurement error

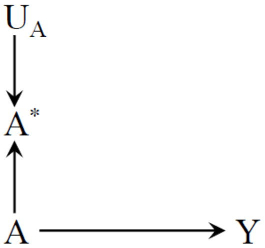


**Figure 9.1**

- $A$: cholesterol-lowering drug
- $Y$: liver disease
- $A^*$: measured use of drug
- $U_A$: physician's memory, patient's attendance

# Measurement error

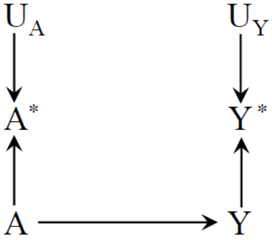


**Figure 9.2**

- outcome $Y$ can be measured with error too
- $Y^*$: measured outcome
- $U_Y$: measurement error for $Y$

# Measurement error

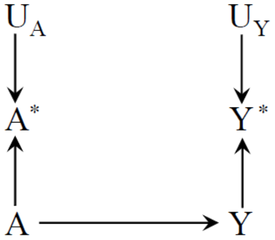


**Figure 9.2**

- no confounder and selection bias

  → *association is causation*

$$\frac{Pr\left[Y = 1|A = 1\right]}{Pr\left[Y = 1|A = 0\right]} = \frac{Pr\left[Y^{a=1} = 1\right]}{Pr\left[Y^{a=0} = 1\right]}$$

# Measurement error

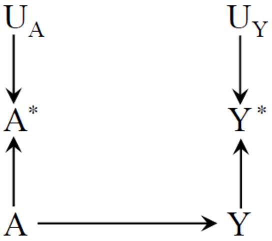


**Figure 9.2**

- In realistic setting, measure of association between $A^*$ and $Y^*$ differ from that of $A$ and $Y$

$$\frac{Pr\left[Y^* = 1 | A^* = 1\right]}{Pr\left[Y^* = 1 | A^* = 0\right]} \neq \frac{Pr\left[Y^{a=1} = 1\right]}{Pr\left[Y^{a=0} = 1\right]}$$

$\rightarrow$ **measurement bias**

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## The structure of measurement error

- Causal structure of
  - *confounding*: presence of common causes
  - *selection bias*: conditioning on common effects
  - **measurement bias**: *independence*, *nondifferentiality*

# The structure of measurement error

- $U_A$ and $U_Y$ are *independent* $\Leftrightarrow f(U_Y, U_A) = f(U_Y)f(U_A)$
- $U_A$ is *nondifferential* $\Leftrightarrow f(U_A|Y) = f(U_A)$
- $U_Y$ is *nondifferential* $\Leftrightarrow f(U_Y|A) = f(U_Y)$

# The structure of measurement error

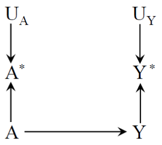



**Figure 9.2**

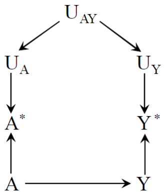



**Figure 9.3**

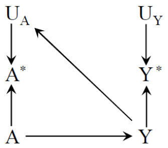



**Figure 9.4**

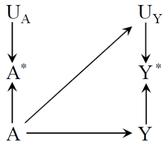



**Figure 9.5**

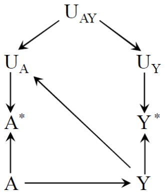



**Figure 9.6**

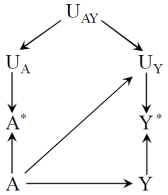
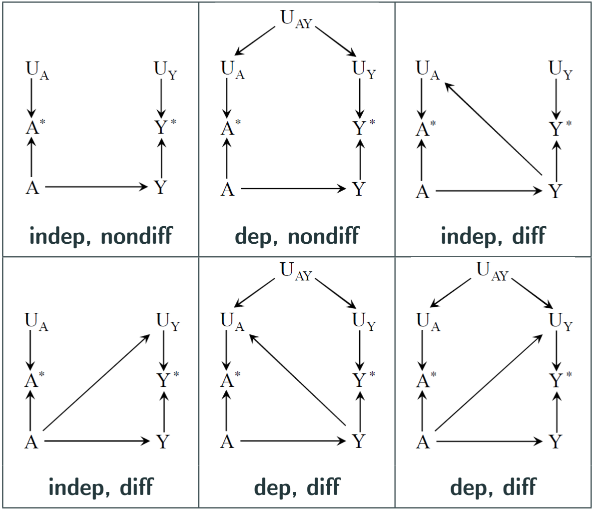



**Figure 9.7**

# The structure of measurement error




- Structure determines the error correction method (not in this book)

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# Mismeasured confounders

- Mismeasurement of confounders will result in bias even if both treatement and outcome are perfectly measured

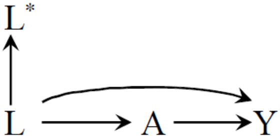


**Figure 9.8**

- $A$: drug use
- $Y$: liver disease
- $L$: history of hepatitis

# Mismeasured confounders

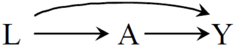


**Figure 7.1**

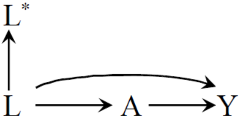


**Figure 9.8**

- In figure 7.1
  - open backdoor path $A \leftarrow L \rightarrow Y$: confounding
  - IP weighting or standardization to compute the average causal effect
- In figure 9.8
  - if the confounder $L$ was not perfectly measured...
  - the backdoor path cannot be blocked by conditioning on $L^*$

# Mismeasured confounders

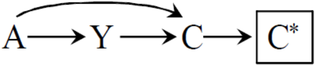


**Figure 9.10**

- It is also possible that a collider $C$ is measured with error
- $C^*$ is a common effect of $A$ and $Y$
  $\rightarrow$ conditioning on $C^*$ will intoduce selection bias

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# Intention-to-treat effect

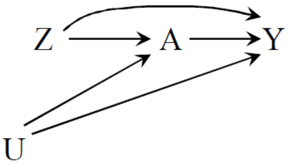


**Figure 9.11**

- $Z$: *assigned treatment*
- It is different from the missclassified treatments $A^*$
- $A^*$ does not have a causal effect on $Y$
- association between $A^*$ and $Y$ is entirely due to $A$

# Intention-to-treat effect

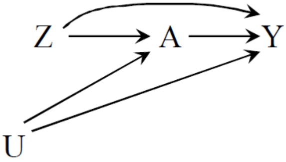


**Figure 9.11**

- The effect of $Z$ does not measure the effect of treating with $A$
- Intention-to-treat effect
  - the effect of assigning participants to being treated with $A$
  - $Z \to A \to Y$, $Z \to Y$

# Intention-to-treat effect

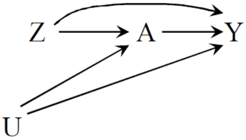


**Figure 9.11**

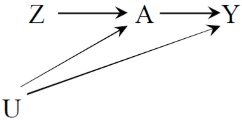


**Figure 9.12**

- Often investigators try to partly decontaminate the effect of $Z$ by eliminating $Z \rightarrow Y$
- *double-blind-placebo-controlled* randomized experiment

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# Per-protocol effect

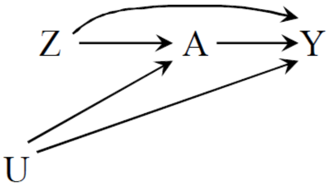


**Figure 9.11**

- Intention-to-treat effect
  - the effect of assigning participants to being treated with $A$
  - $Z \rightarrow A \rightarrow Y$, $Z \rightarrow Y$
- Per-protocol effect
  - the effect of treatment that would have been observed if all individuals had adhered to their assigned treatement
  - $A \rightarrow Y$

## Per-protocol-effect

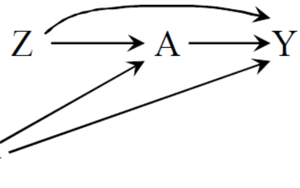


**Figure 9.11**

- $U$: severe illness
- If severely ill individuals in the $Z = 0$ group tend to seek a heart transplant ($A = 1$) outside of the study
- Backdoor path $A \leftarrow U \rightarrow Y$: confounding exists
- However, since $Z$ is randomly assigned, there is no confounding for the effect of $Z$

# Per-protocol-effect

| Intention-to-treat effect | Per-protocol effect |
|---|---|
| Effectiveness | Efficacy |
| may not measure the treatment effect | measure the treatment effect |
| easier to compute | hard to compute (adjustment required) |

- trade-off between bias due to potential unmeasured confounding
- preference that needs to be justified in each application